Seating for PHYS 1020 Midterm
Thursday, October 22
7 - 9 pm

Seating is by last name

<table>
<thead>
<tr>
<th>Room</th>
<th>From</th>
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<tr>
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<td>A</td>
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Chapter 5: Uniform Circular Motion

• Motion at constant speed in a circle

• Centripetal acceleration

• Banked curves

• Orbital motion

• Weightlessness, artificial gravity

• Vertical circular motion
Centripetal Force

Acceleration toward centre of a circular path of radius $r$:

$$a_c = \frac{v^2}{r}$$

Force needed to maintain the centripetal acceleration = centripetal force:

$$\vec{F}_c = m \vec{a}_c = -m \frac{v^2}{r} \hat{r}$$

Force provided by tension in a string, friction, horizontal component of airplane's lift, gravity...

Note: Centrifugal force is the force you feel toward the outside of a curve when going around a corner. It's not really a force, but a consequence of Newton's first law that says that things travel at constant velocity unless a force is applied.
Car A uses tires with coefficient of static friction 1.1 with the road on an unbanked curve. The maximum speed at which car A can go around this curve is 25 \( \text{m/s} \). Car B has tires with friction coefficient 0.85.

What is the maximum speed at which car B can negotiate the curve?

From last time, \( \mathbf{v} = \sqrt{\mu_s rg} \)

So, \( \mathbf{v}_A = \sqrt{1.1 \times 9.8} = 25 \frac{\text{m}}{\text{s}} \)

\( \mathbf{v}_B = \sqrt{0.85 \times 9.8} = ? \)

\( \Rightarrow \) Calculate the ratio

\[ \frac{\mathbf{v}_B}{\mathbf{v}_A} = \sqrt{\frac{0.85 \times 9.8}{1.1 \times 9.8}} = \frac{\mathbf{v}_B}{25} \]

So, \( \mathbf{v}_B = 25 \sqrt{\frac{0.85}{1.1}} = 22 \frac{\text{m}}{\text{s}} \)
Flying around in circles

For no banking, Newton's 1st Law gives

\[ \frac{1}{2} L + \frac{1}{2} L - mg = 0 \]

\[ L = mg \]

The Plane banks to turn in a horizontal circular path of radius \( r \)

\[ L \cos \theta = mg \]

\[ L \sin \theta = m \frac{v^2}{r} \]

So,

\[ \frac{L \sin \theta}{L \cos \theta} = \frac{v^2}{mg} = \frac{v^2}{rg} = \tan \theta \]

This the angle of banking needed to make a turn without gaining or losing elevation!
Problem 5.25

A jet \( (m = 200,000 \text{ kg}) \), flying at 123 m/s, banks to make a horizontal turn of radius 3810 m. Calculate the necessary lifting force.

\[ \theta = \tan^{-1} \left( \frac{123^2}{3810g} \right) = 23.2^\circ \]

So, looking only in the vertical direction:

\[ L \cos \theta = L \cos(23.2^\circ) = mg \]

\[ L = \frac{mg}{\cos(23.2^\circ)} = 2.1 \times 10^6 \text{ N} \]

Also note that:

\[ L^2 \cos^2 \theta + L^2 \sin^2 \theta = L^2 = (mg)^2 + \left( m \frac{v^2}{r} \right)^2 \Rightarrow L = m \sqrt{\frac{v^4}{r^2} + g} \]
Driving around in circles - banked road

(no friction)

As for plane but with lift force replaced by normal force:

Here we have:

vertical direction: \( F_n \cos \theta - mg = 0 \) \( \Rightarrow \) \( F_n \cos \theta = mg \)

horizontal direction: \( F_n \sin \theta = m \frac{u^2}{v} \)

So, again: \( \tan \theta = \frac{u^2}{v g} \), as in the case of the plane.
If you drive slowly, you slide down the slope.

If you drive fast, you skid up the slope.

If $\theta = 31^\circ$ and $r = 316 \text{ m}$ and there is no friction, what is the best speed to drive around the banked curve?

Use the result from last page: $\tan \theta = \frac{v^2}{rg}$

So, $v = \sqrt{rg \tan \theta} = \sqrt{316 \times 9.8 \times \tan (31^\circ)} = 43.1 \text{ m/s} = 155 \text{ km/h}$
Problem 5.20

Two banked curves have the same radius. Curve A is banked at $13°$, curve B at $19°$. A car can travel around curve A without relying on friction at a speed of $18 \text{ m/s}$. At what speed can this car travel around curve B without relying on friction?

\[
\begin{align*}
\omega_A &= \sqrt{rg \tan(13°)} = 18 \frac{\text{m}}{\text{s}} \\
\omega_B &= \sqrt{rg \tan(19°)} = 2 \frac{\text{m}}{\text{s}} \\
\text{So,} \quad \omega_B &= 18 \frac{\text{m}}{\text{s}} \sqrt{\frac{\tan(19°)}{\tan(13°)}} \\
&= 18 \frac{\text{m}}{\text{s}} \frac{\tan(19°)}{\tan(13°)} = 22 \frac{\text{m}}{\text{s}}
\end{align*}
\]
Problem 5.18

A car travels at 28 m/s around a curve of radius 150 m. A mass is suspended from a string from inside the roof.

The force toward the center of the circle is due to the tension in the string, so horizontally we have:

\[ T \sin \Theta = m \frac{v^2}{r} \]

Vertically we have

\[ T \cos \Theta = mg \]

So, again

\[ \tan \Theta = \frac{v^2}{mg} \]

and

\[ \Theta = \tan^{-1} \left( \frac{28^2}{150 \times 8} \right) = 28.1^\circ \]
Orbiting Earth

“The secret to flying is to throw yourself at the earth and miss.”
Hitch Hiker’s Guide to the Galaxy

The centripetal force on the satellite is provided by the gravitational force from the earth.

\[
\vec{F}_c = m \vec{a}_c = -m \frac{v^2}{r} \hat{r} = -\frac{GM_E m}{r^2} \hat{r}
\]

So:

\[
v = \sqrt{\frac{GM_E}{r}}
\]

The smaller the radius, the greater the speed.

Synchronous orbit: period = 24 hours
- satellite stays above same part of earth (above the equator)
- used by communications satellites
- what is the radius of the orbit?